**Project title**  
Aperiodic Cellular Structures  

**Principal supervisor**  
Uwe Grimm  

**Second supervisor**  
Iestyn Jowers and Richard Moat (School of Engineering and Innovation)  

**Discipline**  
Applied mathematics, interdisciplinary  

**Suitable for**  
Full time applicants  

**Project background and description**

Aperiodic crystallographic structures are ordered but are not symmetric under translation. They form a relatively new field of inquiry, and recent discoveries have fundamentally changed our understanding of the mathematics of crystallography and its applications in materials science. The study of aperiodic order has introduced new tilings of space that were previously unimagined, and the subsequent discovery in the 1980s of quasicrystals – material structures that exhibit aperiodic order – has led to explorations of the remarkable mechanical properties of materials that arise from these tiling structures. To date, research on aperiodic structures has mostly been about either the macroscopic scale, e.g. about tilings of space like the famous Penrose tiles, or the microscopic scale, e.g. about arrangements of atoms in quasicrystals. This project is an investigation of aperiodic structures at an intermediate mesoscopic scale, with the aim to incorporate these as internal cellular structures in high-value components, e.g. Figure 1.

Developments in additive manufacturing have made it possible to design and fabricate components for performance, and the inclusion of internal cellular structures puts material in place to carry necessary forces but leaves empty space elsewhere. We anticipate that aperiodic cellular structures will offer improved mechanical performance, and will be of interest in areas such as aerospace, medical engineering, and product design.

The supervision team brings together multidisciplinary expertise from mathematics, design, and materials engineering. We will support an investigation into methods for generating and fabricating structures based on aperiodic order according to external shape constraints, with an aim to demonstrate the mechanical advantage of aperiodic structures over periodic and stochastic structures. These aims align with the following hypotheses:

1. Methods of generative design and fabrication can be used to create aperiodic structures
according to geometric constraints

2. Aperiodic cellular structures will have improved mechanical properties compared to periodic or stochastic structures, and will open new possibilities in design and manufacturing

Methodology
To explore the first hypothesis, the research will investigate algorithms for generating aperiodic frameworks, and address key problems, including creating geometric data models suitable for analysis and fabrication; matching boundaries to required surface shapes; and ensuring rigidity of the structures. Successful validation of the first hypothesis is critical for addressing the second, but we are confident that methods for generating and fabricating aperiodic structures can be found. Uncertainty lies in how efficient these methods can be in terms of autonomy and speed, and how much human intervention is necessary. In particular, autonomously transforming and scaling generated structures to fit specified boundary geometries will be a major challenge. To explore the second hypothesis, the research will involve physical testing of fabricated components. This will enable comparison of the mechanical properties of foams, periodic and aperiodic structures. Analytical models will be constructed based on existing models, and will be applied to predict general properties. But, the complex geometry of the new structures and the uncertainty about resulting material properties mean that the validity of these models will be questioned.

Relevance
The recent emergence and commercial adoption of additive manufacturing technology has introduced new possibilities with respect to the types of shapes that can be realised. In manufacturing, it is now possible to put material only where it is needed, and as a consequence components can be economically designed and fabricated so that they are customised for purpose. This trend, in combination with the drive to improve mechanical properties, has changed the way that components are designed, and in sectors such as aerospace and medical engineering there has been a shift in focus from the external form of components to their internal form. Taking cues from natural porous bone-like materials, components are designed with internal cellular structures that have material in place to carry necessary forces, but empty space elsewhere. These internal structures are on the mesoscopic scale, with solids and voids scaled from 0.1mm-10mm, and are mostly either foams or periodic lattices. Foams are typically manufactured via bubble formation and consequently are stochastic structures. They deliver significant benefits in terms of weight and strength, but are inefficient. Periodic lattices present an improvement in performance and can be designed so that they yield varying mechanical properties, including differential structural properties along axes, e.g. presenting stiffness and rigidity in one direction with a flexibility and mobility along another. But they can be brittle due to the geometry of the structures which allows cracks to propagate catastrophically. Also, periodic structures can be inherently unstable and in order to ensure rigidity extensive bracing is required, resulting in an increase in weight and design complexity.

The project will investigate alternative structures, inspired by nature, and in particular by the atomic arrangements found in novel materials called quasicrystals. Quasicrystals are aperiodically ordered crystals which were first observed in 1982 in rapidly cooled aluminium manganese alloys, an unexpected and fundamental discovery that was recognised by the award of the 2011 Nobel Prize in Chemistry to Dan Shechtman. They are typically intermetallic alloys characterised by point-like diffraction patterns which feature symmetries that are incompatible with periodicity, such as icosahedral symmetry. As a result of their unusual structure, quasicrystals show remarkable properties; they are usually very hard and resemble ceramics rather than metallic materials. Their structure can be more isotropic than
periodic crystals giving nearly uniform mechanical properties in all directions.

A number of promising applications of quasicrystals and quasicrystalline nanoparticles have been reported, but this research explores a different approach; rather than using the actual materials it explores how to employ the underlying structure to generate and fabricate structural frameworks with aperiodic order. To our knowledge, the mechanical properties of frameworks based on aperiodic tilings of space have not been investigated (neither mathematically nor practically), though some tilings have found their way into art and architecture for aesthetic reasons. We anticipate that the symmetries available in aperiodic order can further improve the mechanical performance of internal cellular structures. It is likely that aperiodic structures will overcome the inefficiencies of periodic lattices, because they do not incorporate translational symmetry, and the non-regularity of the geometry means 3-dimensional rigidity is ensured.

The project aims to increase the range of potentially manufacturable structures, by introducing new internal structures for components within defined external envelopes, which improve mechanical properties without affecting other parts of a product. It consolidates UK expertise in the design of cellular structures, presenting a step change in the types of structures manufactured and offers opportunities for new software support and targeted development of manufacturing capability.

**Indication of project timeline**

**Year 1** Investigation and application of the mathematics of aperiodic order, with an aim to generate and fabricate aperiodic structures

**Year 2** Development of algorithms for generating aperiodic frameworks, including creating geometric data models suitable for analysis and fabrication, matching boundaries to required surface shapes, and ensuring rigidity of the structures

**Year 3** Physical testing on fabricated components to enable comparison of the mechanical properties of foams, periodic and aperiodic structures, and validation of existing analytical models